Northern Parade Schools Calculation Policy

At Northern Parade, we want all children to develop a deep conceptual understanding of the formal written methods. The aim is that all children will be fluent in these written methods, and be able to explain the procedures confidently.

Our calculation policy aims to allow children to progress through at their own pace, rather than putting a limit on their learning by stating a year group for each formal method. Children need to be developing a deeper understanding *alongside* learning the process of each method. This may mean that for each new year group objective, they may need to recap a practical or non-standard method before becoming secure with the formal method again. As a Numicon affiliated school, we aim to embed learning using a practical approach for all children.

By the end of year 6, children should be secondary ready, confidently using all formal written methods with a secure understanding of what each operation means e.g. 37 x 4 means 4 lots of 37 and children should know that this can be represented in a variety of ways.

Each operation has been split into 3 areas:

Quick Recall and Derived Facts – these are the facts that children should know instantly, as quickly as they can recall their own name!

Procedures – children need to be fluent in these skills and recognise when to use each method, or recognise when there may be a shorter route e.g. sometimes times tables are quicker to use than short division, counting on might be quicker than completing a column subtraction with lots of 0's etc.

Conceptual Understanding – this shows the children that have a real understanding of how to manipulate numbers using the written procedures.

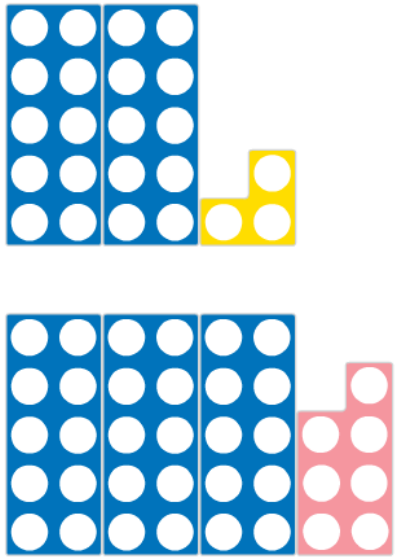
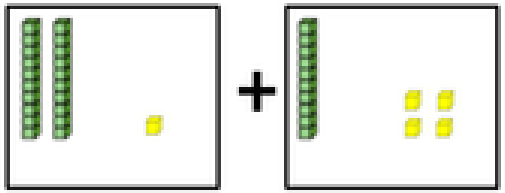
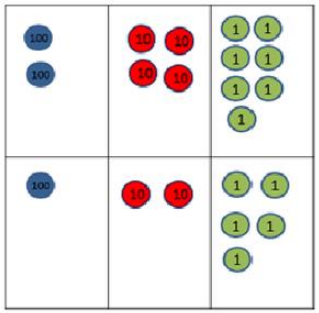
Addition

Addition can appear in 2 different ways:

Aggregation – a question that requires children to add 2 amounts together.

Augmentation – a questions that requires children to make one amount bigger.

To support addition we would use a range of practical apparatus including:

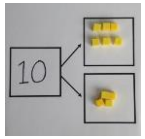
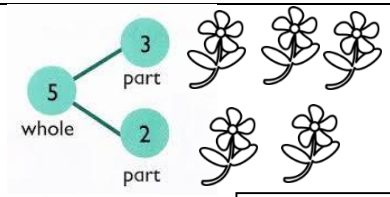
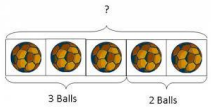
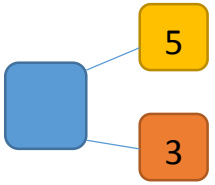
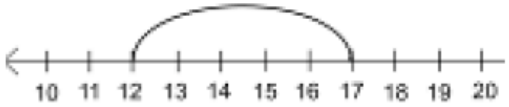
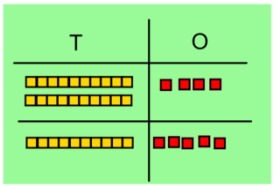
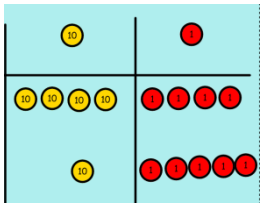
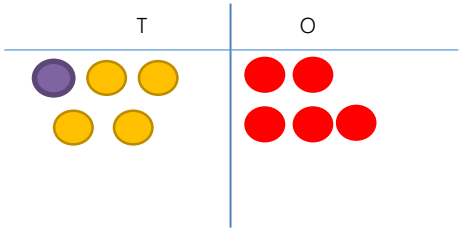
Numicon 23 + 37	Dienes / Base 10 Apparatus 21 + 14	Place Value Counters 247 + 125
		 200 + 40 + 7 100 + 20 + 5 300 + 60 +12 = 372 247 +125 12 60 300 372

We also use a range of other resources such as multilink, straws, bead strings, counters and many more!

Use of concrete, pictorial and abstract

Encourage children to use a variety of ways to record their work.

Below are some examples, although there are many other than can be used:

Concrete	Pictorial	Abstract
   Use cubes to add two numbers together as a group or in a bar. 	  Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	Calculations 21 + 42 = 21 + 42

Quick Recall and Derived Facts

BIG MATHS:
Year 3: - All single digit additions to 20 including doubles
- Add tens e.g. 30 + 40 = 70

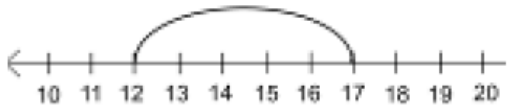
Add hundreds e.g. 300 + 400 = 700
Year 4 : - Add thousands e.g. 3000 + 4000 = 7000
- Add tenths e.g. 0.3 + 0.4 = 0.7

Year 5: - Add hundredths e.g. 0.03 + 0.04 = 0.07
Year 6 : - confidence in mentally adding all of the previous years

In Foundation Stage children explore the concept of counting and amounts. They use manipulatives to combine two amounts and record using numbers and symbols..

Numbered Number Line

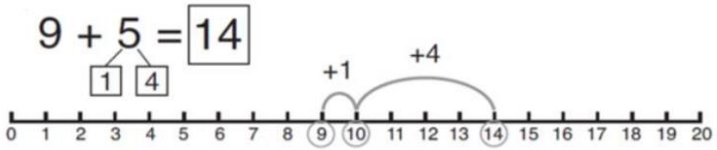
Children can use a variety of resources to support their learning using the number line: Numicon, beads, tracks & Cuisenaire, multilink.



Start at the larger number on the number line and count on in ones or in one jump to find the answer.

Encourage children to count on in ones and develop confidence before moving onto bigger jumps.

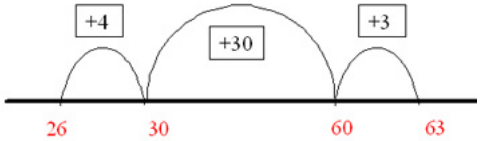
Encourage use of number bonds when bridging to 10.

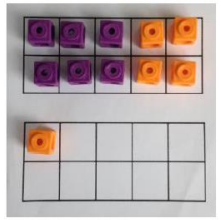


Empty Number Line

Children continue to use a variety of resources to support their learning using the empty number line.

Same strategies as numbered number line but children are expected to draw their own number line and know the starting point. Reinforce use of number bonds and regrouping wherever possible.





Using two ten grids as shown right can help children quickly see these links to support learning.

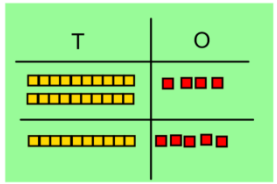
NOTE: Number lines can also be positioned vertically – this helps develop understanding, sometimes making it clearer for children to see, and prepares for learning about negatives later on in curriculum.

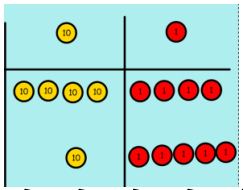
Addition Using Place Value - Before moving onto column methods children should move from number line to addition using place value.

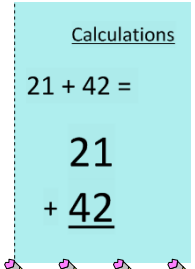
Practical

24 + 15=

Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. Children once confident may progress onto drawing their resources onto a place value grid and begin to record in a written format:







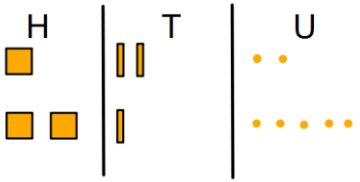
To support place value learning, children may label the columns. Children are not expected at this stage to solve without resources, the written way is simply shown as a way of recording what they have done.

Column Methods

Children should be secure with the process of column addition at each stage before exchange is introduced.

Expanded Column Method – could be practised with dienes, straws or place value counters

Practical (dienes):



122 + 215 = 337

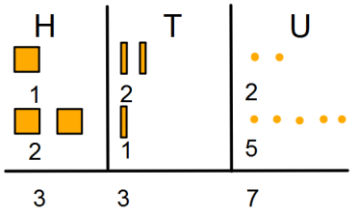
Written:

H	T	U
100	20	2
200	10	5
300	30	7

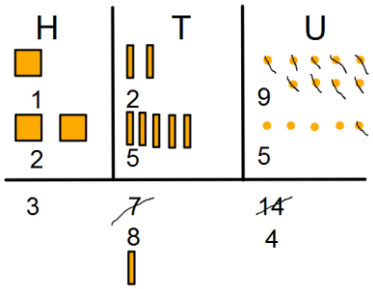
Expanding the column method allows the children to the relationship between the numbers in each column, the value of each digit and the exchange process.

Standard Written Method - Column Addition

Practical (dienes)



(without exchange)



(with exchange)

Written:

```
  7648
+ 1486
  9134
  1 11
```

```
    42
  6432
   786
     3
+ 4681
 11944
  1 2 1
```

When they are ready, children may draw their own dienes before they are confident to use the standard written method.

Conceptual Understanding

Write in the missing digits to make this correct.

	6		8
+	3		9	
9	0	1	9

Convince me

The total is 201. Each missing digit is either a 9 or a 1. Is there only one way of doing this or lots of ways?
Convince me!

Subtraction

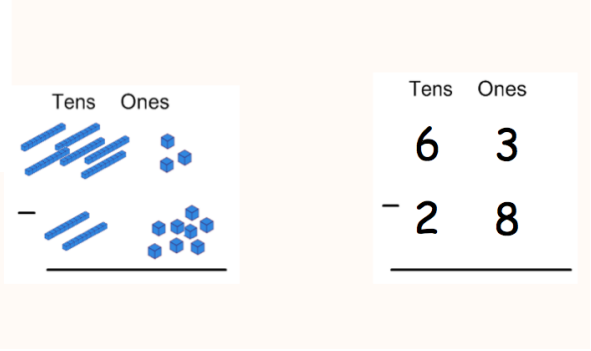
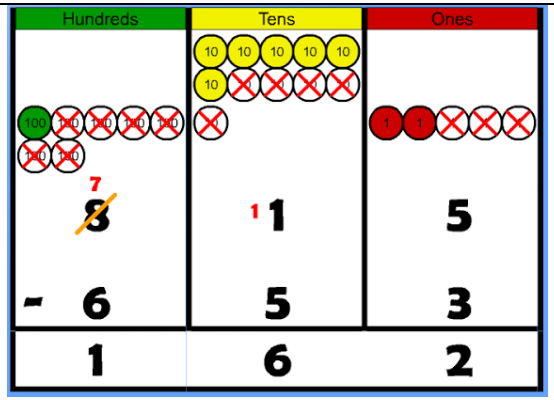
Subtraction can appear in 3 different ways:

Reduction / Take-Away – a question that requires children to remove part of an amount

Comparison – a question that requires children to compare 2 amounts to see which is bigger – "how many more…"

Difference – a question that requires children to find out the numerical difference between 2 amounts

To support subtraction we would use a range of practical apparatus including:

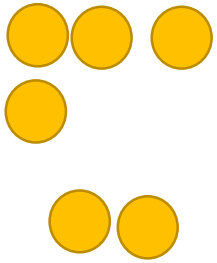
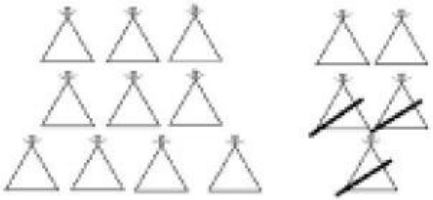
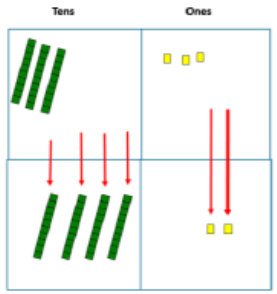
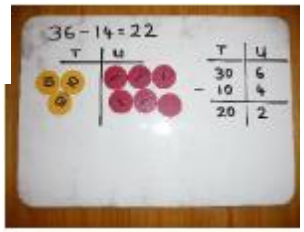
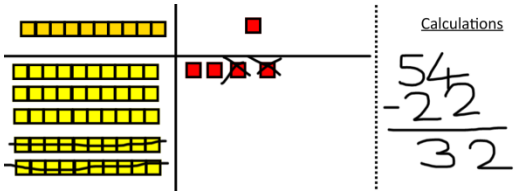
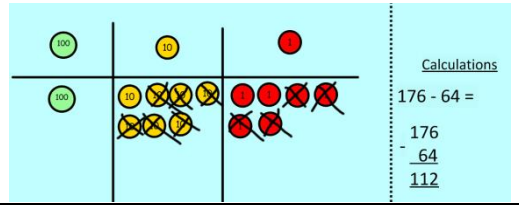
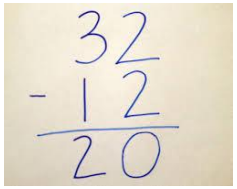
Numicon 9 - 4	Dienes / Base 10 Apparatus 63 - 28	Place Value Counters 815 - 653
		

We also use a range of other resources such as multilink, straws, bead strings, counters and many more!

Use of concrete, pictorial and abstract

Encourage children to use a variety of ways to record their work.

Below are some examples, although there are many other than can be used:

Concrete	Pictorial	Abstract
Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4 	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
 Show how you partition numbers to subtract. Again make the larger number first.	Use Base 10 to make the bigger number then take the smaller number away.   Draw the Base 10 or place value counters alongside the written calculation to help to show working. 	This will lead to a clear written column subtraction. 

Quick Recall and Derived Facts

BIG MATHS:

Year 3: - All single digit subtractions to 20 (understanding the related links to addition)

- Subtract tens e.g. 70 – 30 = 40
- Subtract hundreds e.g. 700 – 300 = 400

Year 4 : - Subtract thousands e.g. 7000 – 3000 = 4000

- Subtract tenths e.g. 0.7 – 0.3 = 0.4

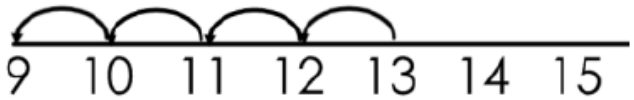
Year 5: - Subtract hundredths e.g. 0.07 – 0.03 = 0.04

Year 6: Mentally subtract a range of numbers with differing decimal places

In Foundation stage children explore how an amount decreases and use manipulatives to physically remove items. They begin to use the language of subtraction and record using symbols and numbers.

<u>Numbered Number Line</u>

Children can use a variety of resources to support their learning using the number line: Numicon, beads, tracks & Cuisenaire, multilink.



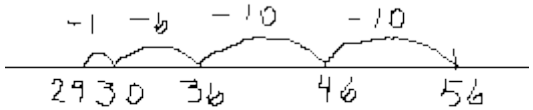
Start at the bigger number and count back the smaller number showing the jumps on the number line.

Children should jump in ones to gain confidence before taking away larger numbers. Link in number facts where possible to help develop mental subtraction strategies and flexibility with numbers. For example, 13 – 4, subtract 3 first to get you to 10, then 1.

<u>Empty Number Line</u>

Children can use a variety of resources to support their learning using the number line: Numicon, beads, tracks & Cuisenaire, multilink.

Children are expected to draw their own numberline, recognising the starting point for the subtraction. Children should jump in ones to gain confidence before taking away larger numbers at a time. Link to number facts where possible to help develop mental strategies and flexibility with numbers.



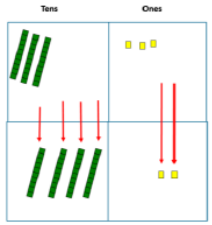
56 - 27

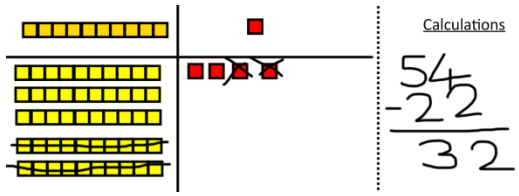
NOTE: Number lines can also be positioned vertically – this helps develop understanding, sometimes making it clearer for children to see, and prepares for learning about negatives later on in curriculum.

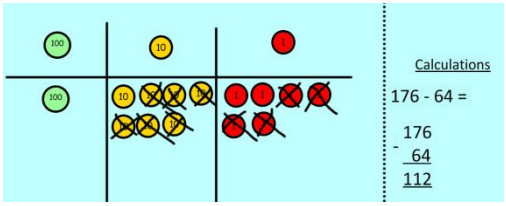
<u>Subtraction Using Place Value</u>

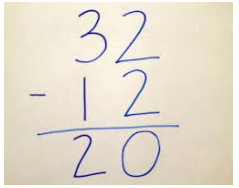
Using a place value grid and resources, make the bigger number then physically take the smaller number away.

Once children are confident with this they may then progress to drawing their resources using a place value grid and begin to annotate as they are working. This will lead to a clear written column subtraction where children may need to label each column with Tens and Ones. Children are not expected to do all calculating in their head at this stage, resources still play an important part.









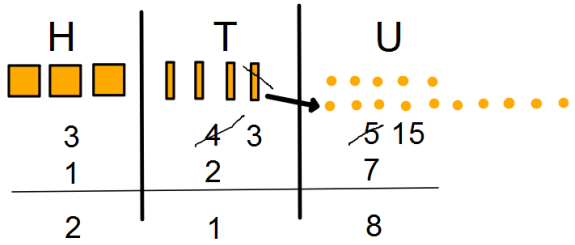
Progress in subtraction procedures

Children should be secure with the process of column subtraction at each stage before exchange is introduced.

Expanded Column Method

Practical Column Method

345 – 127



345 - 127 = 218

Completing a subtraction practically allows the children to understand the affect in each column. This is particularly important for exchanging.

Expanded Written Method

754 –
– 86

Step 1 700 + 50 + 4
– 80 + 6

Step 2 700 + 40 + 14 *(adjust from T to U)*
– 80 + 6

Step 3 600 + 140 + 14 *(adjust from H to T)*
– 80 + 6
600 + 60 + 8 = 668

This would be recorded by the children as

600 140
~~700~~ + ~~50~~ + 14
– 80 + 6
600 + 60 + 8 = 668

Standard Written Method: Column Subtraction

874 – 523 becomes

	8	7	4
–	5	2	3
	3	5	1

Answer: 351

without exchange

932 – 457 becomes

8 12 1
~~9~~ ~~3~~ 2
– 4 5 7
4 7 5

Answer: 475

with exchange

Conceptual Understanding

Subtracting to 2008

In this subtraction, *P*, *Q*, *R* and *S* are digits. What is the value of *P*+*Q*+*R*+*S*?

8 Q 0 S
– P 0 R 2
2 0 0 8

Convince me

□ – 666 = 8□5

What is the largest possible number that will go in the rectangular box?

What is the smallest?

Convince me

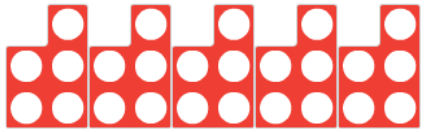
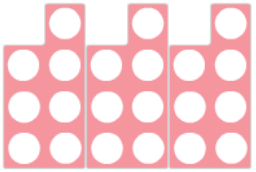
Multiplication

Multiplication can appear in 2 different ways:

Scaling– a question that requires children to increase an amount a number of times

Group Size – a question that requires children to repeated count groups of the same amount

To support multiplication we would use a range of practical apparatus including:

Numicon	Cuisennaires 3 x 3 The green rod has been increased 3 times.
 5 x 5 5 groups of 5  7 x 3 3 groups of 7	

We also use a range of other resources such as multilink, straws, bead strings, counters and many more!

Quick Recall and Derived Facts

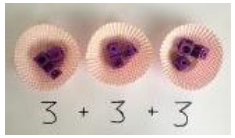
BIG MATHS:
Year 3 – 2, 5, 10, 3, 4, 8, Times Tables
Year 4 – All times tables should be secure and fluent

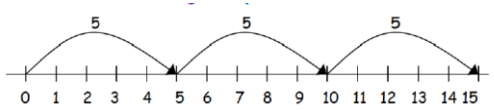
Year 5 – children should be securely using their times table knowledge to derive related fact families
Year 6 – children should be able to use times tables and division facts to derive related facts such as 0.3 x 0.4

Progress in multiplication procedures
Children should be secure with the process of long and short multiplication before being moved on to exchanging.

Repeated Addition

This can be completed using resources and adding equal groups, progressing onto representing using a number line and writing corresponding number sentences. Include multiplication sentences so for 5 + 5 + 5 = 15, I have 3 lots of 5, so my multiplication sentence is 5 x 3 = 15. Once commutativity is introduced using arrays the order of the numbers being multiplied becomes less important.





5 + 5 + 5 = 15

Arrays

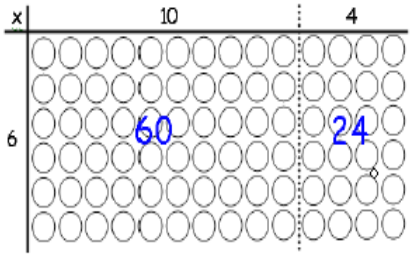
4 x 6 = 24

6 x 4 = 24

Arrays are a cohesive way to show repeated groups, but they also show the commutative property

e.g. 4 x 6 = 6 x 4

For larger multiplications:



(6 x 10) + (6 x 4)

60 + 24

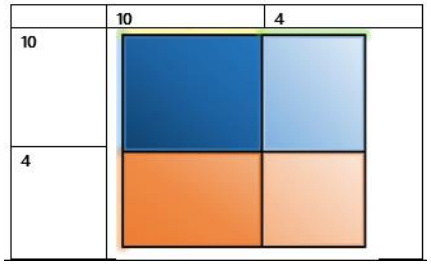
84

This helps children to see that larger multiplications can be partitioned into 'easier chunks'.

Area Grid Method

The area grid method is a step on from arrays, allowing children to visualise the array that would have been there without having to draw out the entire array.

14 x 14



Grid Method

123 x 5

x	100	20	3
5	500	100	15

500
+ 100
+ 15
615

Short Multiplication

This needs to be 'opened out' so that children understand how this has developed from the area grid method, before being condensed down and exchanging numbers across each column.

```
  38
x  7
  56
 210
 266
```

The expanded method can be carried out next to the grid method to allow children to see where each multiplication arises.

Once children can confidently explain the procedure, they will be ready to move to the formal written method.

342 × 7 becomes

	3	4	2
×			7
2	3	9	4
	2	1	

Answer: 2394

2741 × 6 becomes

	2	7	4	1
×				6
1	6	4	4	6
	4	2		

Answer: 16 446

Long Multiplication

Again, it is key that the children understand the procedure happening. It may be necessary to return to one of the previous methods to ensure they are confident.

124 × 26 becomes

		1	2	
	1	2	4	
×		2	6	
2	4	8	0	
	7	4	4	
3	2	2	4	
1	1			

Answer: 3224

124 × 26 becomes

		1	2	
	1	2	4	
×		2	6	
	7	4	4	
2	4	8	0	
3	2	2	4	
1	1			

Answer: 3224

Conceptual Understanding

Prove It

What goes in the missing box?

x	?	?
4	80	12

Prove it.

How close can you get?

 X 7

Using the digits 3, 4 and 6 in the calculation above how close can you get to 4500? What is the largest product? What is the smallest product?

Fill in the missing boxes

		4	□
×	□		6
	2	4	6
	8	2	0
1	0	6	6

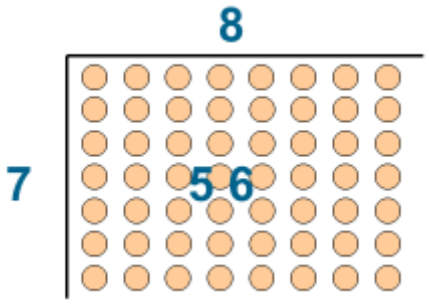
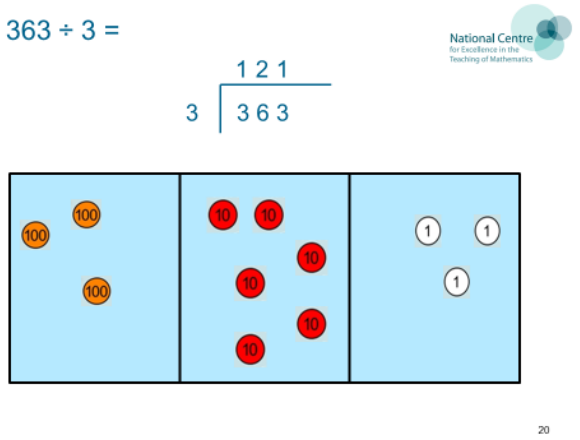
Division

Divisionn can appear in 2 different ways:

Sharing– a question that requires children to share out an amount into a set number of group

Grouping – a question that requires children to put an amount into set group sizes

To support division we would use a range of practical apparatus including:

Straws	Counters 56 ÷ 7	Place Value Counters
		

We also use a range of other resources such as multilink, straws, bead strings, counters and many more!

Quick Recall and Derived Facts

BIG MATHS: Year 3 – 2, 5, 10, 3, 4, 8, Times Tables division facts Year 4 – All times tables division facts should be fluent and secure	Year 5 – children should be securely using their times table knowledge to derive related fact families Year 6 – children should be confident to use times tables facts to explore division with decimals

Progress in Division

Children need to be clear of the 2 different structure in division, and the context that these might be seen in.

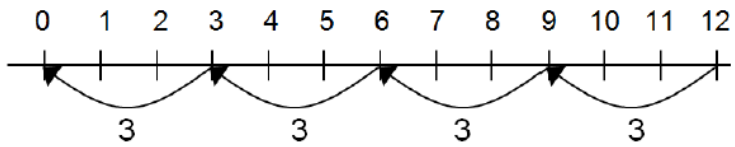
These need to be clear, before formal written methods are introduced.

Grouping Structure

Ensure division sentence is written to correspond with procedure.

As repeated subtraction:

Use a number line to show jumps in groups. The number of jumps equals the number of groups.

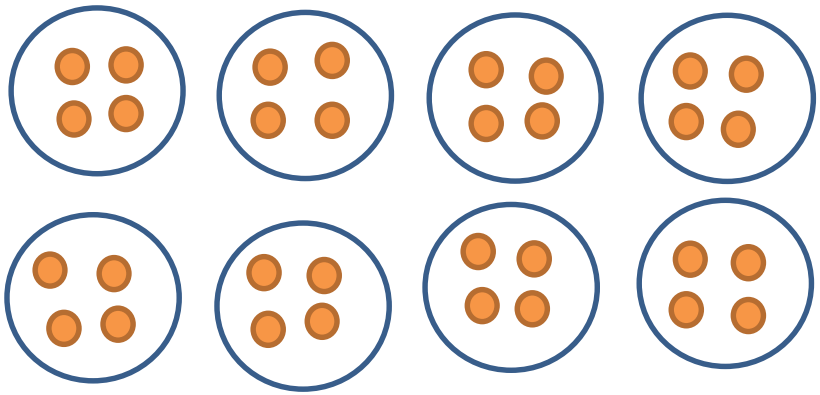


Using resources:

This links closely to times tables and arrays. Children need to understand that division can sometimes be asking:

'If it put sweets into *groups of 4*, how many groups would I get with 32 sweets?'

32 ÷ 4 = 8



The objects have been organised into groups of 4 leaving 8 groups.

Sharing Structure

Using resources or pictures to model sharing with corresponding number sentence.

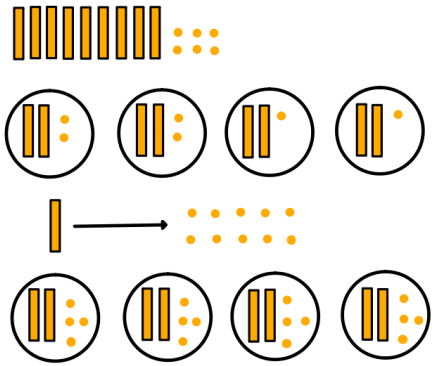
I have 10 cubes, can you share them equally in 2 groups?

10 ÷ 2 = 5

This structure is about sharing out are set of objects, and children need to understand the sharing structure is often seen when asked:

'I have 96 sweets, how many would 4 friends get?'

96 ÷ 4 = 24

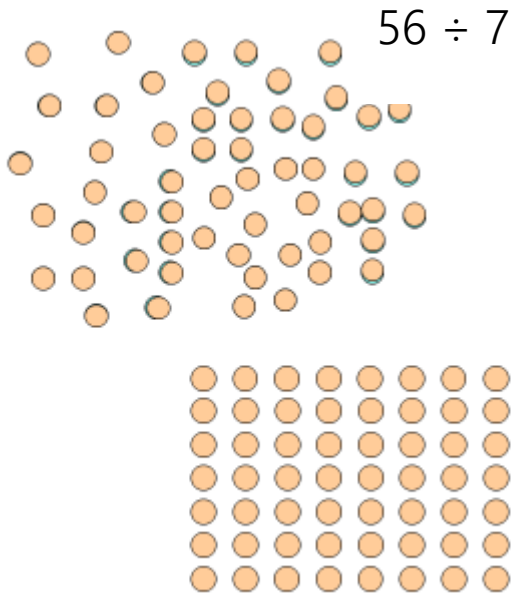


Share out the tens sticks evenly, when they can't be shared out equally, the ten needs to be exchanged for ten ones.

Progress in Division

Written methods should only be introduced and used when children have a really clear understanding of the concept of division, and are securely using resources to show this.

Arrays



Either:

- How many 7s can I see? (grouping)

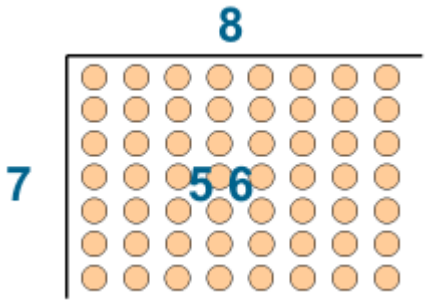
Or:

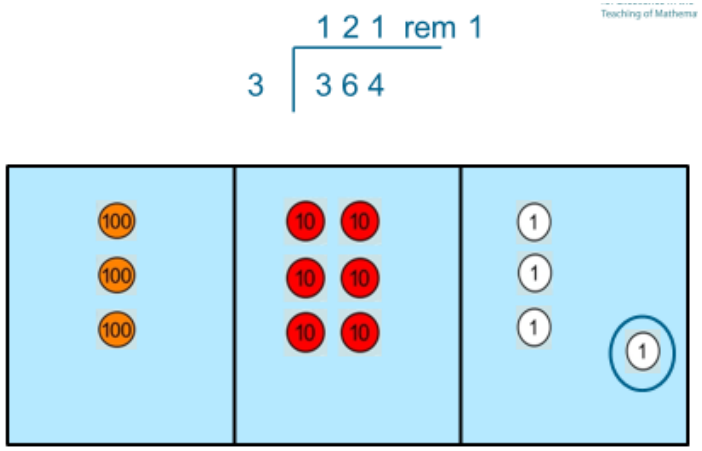
- If I put these into 7 groups how many in each group? (sharing)

Once children are confident with the two different structures, and have explored these in a variety of ways, they need to begin to organise their working coherently.

To organise their working, the first step is to use arrays. These clearly show children the link to multiplication and their times table knowledge.

This can then lead to the beginning of short division.

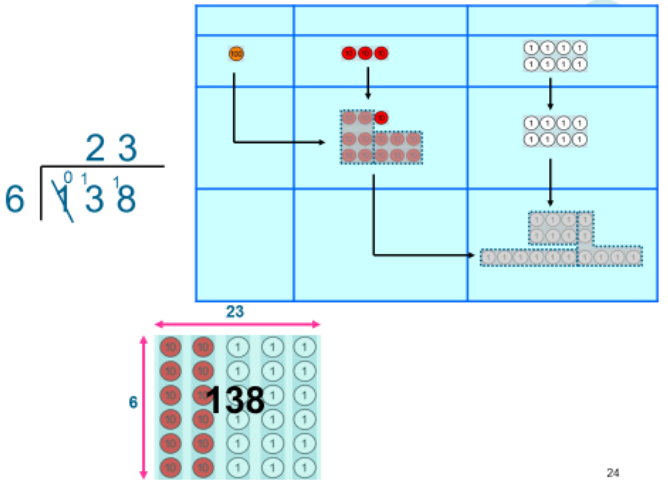




As the numbers begin to get larger, using counters will become inefficient. Place value counters enable children to practise procedural fluency with larger numbers.

The place value counters allow larger numbers to be manipulated while still practising the short written method.

It allows children to see how the number has been divided, while still recognising the value of each digit.
When children are ready, the exchange process can be demonstrated.



Annotated Long Division - leading to Long Division

$$
\begin{array}{r}
24 \\
4\overline{)96} \\
80 \quad (20\times4=80) \\
\hline
16 \\
16 \quad (4\times4=16) \\
\hline
0
\end{array}
$$

Children record the answer as they go rather than 'chunking' to add at the end. This may not be necessary if children have rehearsed the process using practical resources.

432 ÷ 15 becomes

$$
\begin{array}{r}
28 \text{ r } 12 \\
15\overline{)432} \\
300 \\
\hline
132 \\
120 \\
\hline
12
\end{array}
$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$
\begin{array}{rl}
28 & \\
15\overline{)432} & \\
300 & 15\times20 \\
\hline
132 & \\
120 & 15\times8 \\
\hline
12 &
\end{array}
$$

$\frac{12}{15} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

$$
\begin{array}{r}
28\cdot8 \\
15\overline{)432\cdot0} \\
300 \\
\hline
132 \\
120 \\
\hline
120 \\
120 \\
\hline
0
\end{array}
$$

Answer: 28·8

Short division

98 ÷ 7 becomes

$$
\begin{array}{r}
14 \\
7\overline{)98}
\end{array}
$$

Answer: 14

496 ÷ 11 becomes

$$
\begin{array}{r}
45 \text{ r } 1 \\
11\overline{)496}
\end{array}
$$

Answer: $45\frac{1}{11}$

432 ÷ 5 becomes

$$
\begin{array}{r}
86 \text{ r } 2 \\
5\overline{)432}
\end{array}
$$

Answer: 86 remainder 2

Conceptual Understanding

Prove It

What goes in the missing box?

12 ☐ 2 ÷ 6 = 212

14 ☐ 4 ÷ 7 = 212

22 ☐ 3 ÷ 7 = 321 r 6

Prove it

Digital Division

Consider all three-digit numbers formed by using *different* combinations from 0, 1, 2, 3 and 5. How many of these numbers are divisible by 6?